Exercise 16

Prove the identity.

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Solution

Use the definitions listed on page 259.

$$\cosh^2 x + \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{(e^x + e^{-x})^2}{4} + \frac{(e^x - e^{-x})^2}{4}$$

$$= \frac{(e^x + e^{-x})^2 + (e^x - e^{-x})^2}{4}$$

$$= \frac{(e^{2x} + 2e^x e^{-x} + e^{-2x}) + (e^{2x} - 2e^x e^{-x} + e^{-2x})}{4}$$

$$= \frac{2e^{2x} + 2e^{-2x}}{4}$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

$$= \cosh 2x$$